Nonlinear regression model

Statistics and Big Data

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Course: Statistics and Big Data

Overview

What is Logistic Regression?

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How do we predict outcomes that are categorical, such as whether a mouse is obese or not? What if we could model the probability of an event occurring rather than predicting a continuous value?

Linear Regression Recap

Let's consider a familiar example: linear regression. Imagine we have data on mouse weight and size. By fitting a line to this data, we can:

1. Calculate the correlation (R-squared).
2. Determine statistical significance (p-value).
3. Predict size based on weight.

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This process illustrates how linear regression serves as a foundational machine learning technique.

Transition to Logistic Regression

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Now that we understand linear regression, how does logistic regression differ? Instead of predicting a continuous outcome, logistic regression predicts a binary outcome: true or false, obese or not obese.

Core Concept of Logistic Regression

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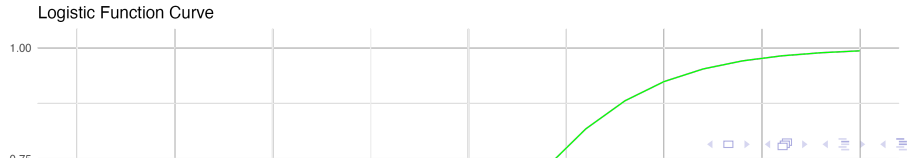
Logistic regression fits an S-shaped curve, known as the logistic function, to the data. This curve represents the probability of an event occurring, ranging from 0 to 1.

Mathematical Formulation

$$P(Y = 1|X) = \frac{1}{1 + e^{-(\beta_0 + \beta_1 X)}}$$

Where:

- $P(Y = 1|X)$ is the probability of the event occurring.
- β_0 is the intercept.
- β_1 is the coefficient for the predictor variable X.



Interpreting the Logistic Function

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What does the logistic function tell us about mouse obesity?

- A heavy mouse has a high probability of being obese.
- An intermediate weight mouse has a 50% chance.
- A light mouse has a low probability.

This probabilistic interpretation is crucial for classification.

Classification with Logistic Regression

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How do we classify a mouse as obese or not? If the predicted probability exceeds 50%, we classify it as obese; otherwise, it is classified as not obese. This classification process can be extended to multiple predictors, such as weight, genotype, and age.

Testing Variable Significance

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How do we determine which variables are useful in our model? We utilize Wald's test to assess whether the effect of each variable is significantly different from zero.

Mathematical Formulation

$$W = \frac{\hat{\beta}}{\text{SE}(\hat{\beta})}$$

Where:

- $\hat{\beta}$ is the estimated coefficient.
- $\text{SE}(\hat{\beta})$ is the standard error of the coefficient.

This helps us identify non-contributing variables, such as astrological sign.

Maximum Likelihood Estimation

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Mathematical Formulation

$$L(\beta) = \prod_{i=1}^{n} P(Y_i|X_i)^{Y_i}(1 - P(Y_i|X_i))^{1-Y_i}$$

Where:

- $L(\beta)$ is the likelihood function.
- Y_i is the observed outcome.

This method finds the parameter estimates that maximize the likelihood of the observed data.

Summary of Logistic Regression

In summary, logistic regression allows us to classify outcomes using both continuous and categorical predictors.

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Key takeaway

Logistic regression is a powerful tool in both traditional statistics and machine learning.

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- **Exercise 3:** Identify which variables might be significant predictors of obesity in a hypothetical study and justify your choices.
- **Exercise 4:** Discuss the implications of using maximum likelihood estimation in logistic regression compared to least squares in linear regression.