Hypotheses testing on an average

Statistics and Big Data

Niccolò Salvini, PhD

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Overview

What is Hypothesis Testing?

Concept

Imagine a scenario where we are testing the effectiveness of two different drugs on recovery time from a virus. How do we determine if one drug is truly better than the other? This leads us to the concept of hypothesis testing, a fundamental aspect of statistical analysis.

Concrete Example of Drug Testing

Drug A Recovery Times

We administer Drug A to three patients and measure their recovery times:

- Patient 1: 10 hours
- Patient 2: 15 hours
- Patient 3: 12 hours

The average recovery time for Drug A is calculated as follows:

$$\text{Mean}_A = \frac{10 + 15 + 12}{3} = 12.33 \text{ hours}$$

Concrete Example of Drug Testing (B)

Drug B Recovery Times

Now, we do the same for Drug B:

- Patient 1: 25 hours
- Patient 2: 20 hours
- Patient 3: 22 hours

The average recovery time for Drug B is:

$$\text{Mean}_B = \frac{25 + 20 + 22}{3} = 22.33 \text{ hours}$$

Formulating a Hypothesis

Hypothesis

Based on our preliminary data, we might hypothesize: **Hypothesis (H1):** Patients taking Drug A recover, on average, 10 hours faster than those taking Drug B.

Unexpected Results from Repeated Experiments

New Averages

Upon repeating the experiment, we find:

- New average for Drug A: 25 hours
- New average for Drug B: 20 hours

This leads to a new average difference of:

$$\text{Difference} = \text{Mean}_B - \text{Mean}_A = 20 - 25 = -5 \text{ hours}$$

Rejecting the Hypothesis

Conclusion

After multiple experiments yielding inconsistent results, we reach a critical conclusion: We can confidently **reject** the initial hypothesis that Drug A is superior. This illustrates the importance of rigorous testing in hypothesis formulation.

Introducing the Null Hypothesis

Definition

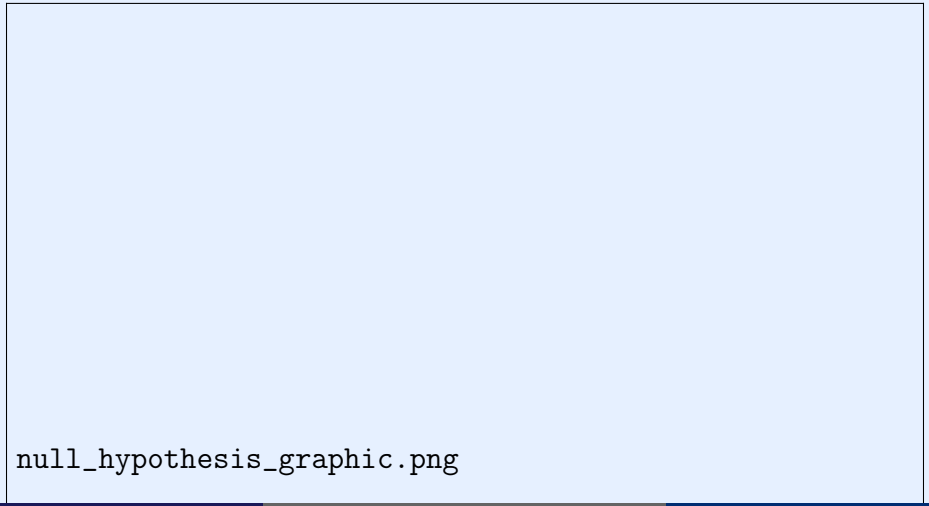
Now, let's introduce a key concept: the **Null Hypothesis (H0)**. The null hypothesis states that there is no effect or difference between the groups being tested.

Null Hypothesis

In our case, we would state: **H0:** There is no difference in recovery time between Drug A and Drug B.

Visualizing the Null Hypothesis

Concept Illustration



Testing the Null Hypothesis

Analysis

When we conduct our experiments, we analyze the data to determine if we can reject the null hypothesis. If the data shows a significant difference that cannot be attributed to random chance, we reject H0.

Summary of Key Concepts

Summary

To summarize:

1. We formulate hypotheses based on preliminary data.
2. We conduct experiments to test these hypotheses.
3. We use the null hypothesis as a baseline for comparison.
4. We reject or fail to reject the null hypothesis based on statistical evidence.

Exercises

Exercise 1

Define the null hypothesis in your own words and provide an example related to drug testing.

Exercise 2

Given the following recovery times for Drug C (8, 9, 10 hours) and Drug D (15, 14, 16 hours), calculate the means and formulate a hypothesis.

Exercise 3

Discuss a scenario where failing to reject the null hypothesis could lead to incorrect conclusions.

Exercise 4

Consider a situation where you have multiple hypotheses. How would you determine