Hypothesis Testing and The Null Hypothesis

Statistics and Big Data

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Course: Statistics and Big Data

Overview

What is Hypothesis Testing?

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Concrete Example of Drug Testing

Let's consider a practical scenario:

Drug A Recovery Times

- Patient 1: 10 hours
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- Patient 3: 12 hours

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- Patient 5: 18 hours
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What do these recovery times suggest about the effectiveness of the drugs?

Observing Differences

From our example, we observe that:

$$\bar{x}_A = \frac{10 + 15 + 12}{3} = 12.33 \text{ hours},$$
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This indicates that patients taking Drug A recover faster on average. But can we conclude that Drug A is definitively better than Drug B?

Formulating a Hypothesis

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The Role of Random Variation

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This variability raises an important question: How can we be sure of our hypothesis?

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Rejecting the Hypothesis

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Key Understanding

If data consistently contradicts our hypothesis, we must reject it.

Introducing New Drugs

Now, let's consider two new drugs, C and D. We hypothesize that:

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Patients taking Drug C recover, on average, 13 hours faster than those taking Drug D.

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What happens when we test this hypothesis?

Testing the New Hypothesis

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In this case, we cannot confidently reject the hypothesis that Drug C is better than Drug D. Instead, we "fail to reject" the hypothesis, indicating that while the evidence is not strong enough to confirm it, it is also not disproven.

Understanding the Null Hypothesis

To clarify our testing process, we introduce the **Null Hypothesis (H0)**:

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Why is this important?

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Summary of Key Concepts

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What are the implications of these concepts in real-world applications?

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